Degradation of a Complex System Modelling

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Abstract: We examine the logical probabilistic modelling of a complex system's blocks failures with the considerations of the connections between the blocks, based on the logical linguistic approach. We describe the modelling procedure, implementing the logical probabilistic and logical linguistic modelling approaches. We developed a model describing a simplified solution of accounting the blocks' connections.

Keywords: complex logical function, complex system maintenance, logical linguistic modelling, logical probabilistic modelling, prognosis,.

I. INTRODUCTION

While any block of the system is used, it experiences the impact of various factors, which leads to altering (worsening) of its technical condition with the passing of time. That leads to failure probability of that block as well as that of the entire system. The peculiarity of those factors is their values' stochastic vibration with the passing of time. The most considerable factors are technological strains, durability characteristics of the unit's material, its geometrical parameters. In addition, it is vital to distinguish the fulfillment of the technological process conditions, the quality of maintenance, repairs, etc. [6]. The mentioned factors are stochastic, and, as such, the time of the failure is also random. Thus, it is suitable to perform the analysis of the complex system failure probability altering with the passing of time and the prediction of its failure via the mathematical and computer modelling.

While modelling, it is common to set the four types of the initial information: the repair statistics, the technological stress data, the resource estimation, and the diagnostic statistics [8]. That allows to divide the existing models to four types, namely resource model, based on the repair dates' data; the force model, based on the durability and the geometrical parameters of the block and the statistics of the technological stresses; diagnostic model, based on the diagnostic data; and expert model, based on the expect estimations of the system blocks' resources. While using any of the mentioned models we have first to define the predicted parameters, and then perform the prediction procedure.

The expert model is the most simple of all the parametrized models. We define its parameters are defined based on the expert estimations of the system blocks' resources. The set of the initial data for this model is given as the expert estimations. Usage of this model is suitable for the early stage of the running of the equipment, when no sufficient statistical information on repairs and technical maintenance is available.

One of the most promising approaches for creating of the expert models is the development of the logical probabilistic methods, which use the functions of algebra of logic (FAL) for analytical views of the system's working conditions and the strict ways of converting FAL to probabilistic functions, which objectively express the reliability of this system [7, 10]. The benefit of the logical probabilistic approach for the engineers is mainly in their high strictness and the vast possibilities for the analysis of every element's impact to the whole system's reliability. However, there are also complexities for the active usage of such methods. Namely, for the complex tasks and structures, which are described by a FAL of any form, it is quite complex to transform the system's description into the probabilistic form.

While transforming the FAL into a Zhegalkin polynomial it is easy to formalize the computation of the probability of the resulting complex logical function (CLF) [1]. However, a CLF describing the failure of a complex system contains a high number of summands. Thus, even more summands will be present in the formula, which computes its probability, since the number of the summands in the expression of the CLF probability is an exponent function of the logical summands' count. It is unlikely that an algorithm for drastic simplifying of exponential computations will be found. In the case of transforming the CLF to orthogonal format (of a perfect disjunctive normal form), the quantity of its summands is also an exponential function of the initial count. Thus, the computing of the probability of the logical function directly, without any initial approximations of the significant summands count leads to very high expends of the processing time or memory [5].

However, we can decrease the amount of computations if we estimate the errors of the summands that are remote from the start of the polynomial, which have a small impact on the computed probability. That lets us to decrease the computations' amounts if we do not take into account the small members of the polynomial of the CLF [5].

For instance, while computing the probability of a CLF polynomial, which contains 45 summands, it is enough to account the first 10 to 15 ones, and then the computation time will be more than 3 times less [2]. In addition, the lexical graphic ordering of the fundamental vector of the CLF ensures the independent computing of the summands in the polynomial expression for its probability, which allows to control the computation process and to make decisions about its terminating while approaching the needed precision [2]. The drawback of such approach is that we need to ensure the independence of the logical variables that we use in the CLF.

We can also make the computing of the complex system failure based on the orthogonal transformations in the algebra of tuples [3]. This is quite a consuming operation; however, there were developed methods of its simplifying [9], which in many cases allow to drastically decrease the computing time for formulae with a high number of variables. For instance, if we use the computation result in numerous subsequent usages with the altering probability values, then the time consumption is justified.

We can considerably reduce the number of computation operations in those cases, when the initial logic formula changes itself due to the changes in the examined system [9]. Furthermore, in the algebra of the tuples we can get the computation formula for precise probability computation even in those cases when the sub formulae in the disjunction are not mutually independent [4]. Yet while estimating the changes in CLF describing system's failure with the passing of time and with the accounting the connections between the blocks (excluding the cases with the simplest schemes), there appear certain complexities and ambiguities [7, 10]. In the current chapter, we examine the possible solutions to the problem of the complex system failure probability modelling with the passing of time with the accounting of connections between the system's blocks.

II. Computing the failure probabilities of a complex system

While using the method of FAL algebraic transformation, described in [6, 7, 10], we can write down the following CLF which describes the failure probability of an n blocks system.

$$\mathbf{Y} = \mathbf{AF} \quad (1)$$

Where A is a rectangular binary matrix containing the identification failure rows $\mathbf{C}_{ij}$, which have the length of $N = 2n - 1$ and consist of ones and zeroes; $\mathbf{F}$ is a fundamental vector of the logical failure system φ_i ($\varphi_i = 1$ means that the ith block is broken) which is also of the length $N = 2n - 1$.

The fundamental vector F is the ordered set of the elements of the Cartesian multiplication of the basis vector of the system's blocks failures:

$$\mathbf{\Phi}^{\mathrm{T}} = \langle \varphi_1, \varphi_2, ..., \varphi_n \rangle \quad (2)$$

Thus

$$\mathbf{F}^{\mathrm{T}} = \langle \varphi_1, \varphi_2, ..., \varphi_n, \varphi_1\varphi_2, \varphi_1\varphi_3, ..., \varphi_{n-1}\varphi_n, ..., \varphi_1\varphi_2\varphi_3, ...\varphi_{n-1}\varphi_n \rangle \quad (3)$$

The distribution of zeroes and ones in the identification failure rows $\mathbf{C}_{ij}$ has to represent the physically feasible system failures. So, if $n = 4$ and if we account the failures of only the first and the second system blocks the row should be as follows:

$$\mathbf{C}_j = \langle 1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0 \rangle \quad (4)$$

That means that either $(\varphi_1 = 1 \wedge \varphi_2 = 1)$ or $(\varphi_1 = 1 \vee \varphi_2 = 1)$ and all the other members equal to zero. That means that depending on the failure type $\mathbf{Y}_j$ there will be different combinations in the rows $\mathbf{C}_j$. Then we define the failure type by the following formula:

$$\mathbf{Y}_j = \mathbf{C}_j\mathbf{F} \quad (5)$$

If we know the failure probabilities for the ith system blocks $P_{fi}\{x_i = 1\}$, then if their failures are independent the probability of the jth failure type of the system $P_{fj}\{\mathbf{Y}_j = 1\}$ can be computed approximately, using the following polynomial formula [8]:

$$P_{fj} = (-1)^0 \sum_{\gamma-1}^{r} P_\gamma + (-1)^1 \sum_{\gamma\eta} P_\gamma P_\eta + ... + (-1)^{r-1} \prod_{\gamma-1}^{r} P_\gamma \quad (6)$$

Where r is the length of $\mathbf{Y}_j$ or the number of ones in the row $\mathbf{C}_j$, $j \neq \eta$ are the numbers of the members of the polynomial (5), P_i are the computed or given probability values of the ith member of the polynomial (5). However, we should define the required computation precision ΔP_{fi} and continue the computation process, started from the first member of the polynomial (6), until the delta is lesser than ΔP_{fi}. As the result of the computation, we shall receive the vector of the system failure probabilities

$$P_c^{\mathrm{T}} = \langle P_{c1}, P_{c2}, ..., P_{cj}, ..., P_{cM} \rangle \quad (7)$$

Where M is the number of j-identified system rows.

It is obvious that we must define the system reliability as the maximal probability of all P_{fj}, which corresponds to the identification row $\mathbf{C}_j$, which contains only ones. With the passing of time T of the complex system working (usually $T = \max\{t_{ik}\}$) the probabilities of correct working of its blocks $P_{ci}(t_{ik})$, where t_{ik} is the kth moment of time of the ith block running, decrease with different velocity, also, different blocks may have different t_{ik}. The latter requires the periodical re-computing of all the P_{cj} and re-considering of the current system reliability, while running the system. However, different situations are also possible. For instance, some ith system blocks may have the reserve blocks, which are initiated only by failure signal v_i of the failed block. There may be some jth blocks in the system, which have the fixed running time intervals Δt_{jk}. There may also be sth blocks in the system, which switch on and off by the external signals θ_s, that may be initiated by a human operator. Also, the may be qth blocks, which reliability may be defined not by their medium time before failures and their correct working probability, but rather by their switching on and off number g_{qk}, as, for instance, various switches. For the latter blocks, we must watch such blocks' switching during the system's running time T and, depending on that number, decrease its modelled correct working probability according to a given rule.

Let us assume that the decreasing of the correct working probability $P_{ci}(t_{ik})$ happens, as usual, by the exponential law [1]:

$$P_{ci}(t_{ik}) = \exp(-\alpha_{ik} t_{ik}) \quad (8)$$

Where α_i is the coefficient of the correct work probability, corresponding to the kth moment of the ith block working (for the qth blocks we must substitute t_{ik} with g_{qk} in the equation of the type (8)). We may find the initial coefficient α_i from the equation (8), if we have the medium time before failure t_i for the ith blocks and the probability of the correct work $P_{ci}(t_i)$ in that moment of time. Usually such parameters are given in the blocks' technical documentation. Similarly, we may compute the values q_0 for the qth blocks. Thus the estimation of the current block reliability as well as the whole system's if the blocks' failures are independent, is not complex, since we may assume that the coefficients α_i are not time-dependent.

If we do not know some probability P_i or we are not sure if the system's blocks failures are independent, the given approach may result in considerable errors [3]. In some cases, we may perform the computing of the complex system's failure probability based on the orthogonal transforming and the algebra of the tuples. However, in that case the process of finding the system's FAL is much more complex, as are the computations. We may solve the given problem approximately using the proposed simplified blocks' connections' accounting method.

III. A SIMPLIFIED APPROACH TO THE BLOCKS' CONNECTIONS' ACCOUNTING PROBLEM

If we know only the fact of the connections' existence or absence, that is, we know only the topology, but the connections' characteristics are unknown, we may increase the precision of the system's failure probability changing with the passing of time by the proposed procedure of simplified mutual interconnected blocks' impact and of the values of the failure probabilities of such blocks. That means that we will use the probabilities that are approximately equal to the conditional probabilities in the equation (6).

The increase of the blocks' failure probabilities with the passing of time is primarily caused by the changes in their parameters. For instance, with the passing of time the sizes of the details alter due to friction. That causes the increase in the vibrations' amplitude. Thus, the failure probability of such block increases. Therefore, the failure probability depends on the expected value (EV) of the block's parameter. With the normal distribution, we may describe the mentioned dependency by the following well-known expression (9):

$$P_{fi}(t_{ik}) = \\ = 1 - \Phi((b_i - m_i(t_{ik}))/\sigma_i) + \Phi((-b_i - m_i(t_{ik}))/\sigma_i) = \quad (9) \\ = 1 - P_{ci}(t_{ik})$$

Where bi is the maximal allowed values of the *i*th block parameter, $m_i(t_{ik})$ and σ_i are respectively its expected value and root mean square, $\Phi(x)$ is the Gaussian probability interval, which cannot be expressed via the elementary functions, but there are tables of its values [5] or its approximate expressions in the form of a row with the decreasing members, such as (10):

$$\Phi(x) = \frac{1}{\sqrt{\text{®}}}\left(x - \frac{x^3}{1!3} + ... + \frac{(-1)^l}{l!(2l+1)}x^{2l+1} + ...\right) \quad (10)$$

Since we usually know the initial expected parameter values $m_i(t_0)$ for each block and the values t_i, $P_{ci}(t_0)$, bi, then we can calculate the value σ_i from the expression of type (9) with the usage of either the table of values $\Phi(x)$ [5], or the simplified value of $\Phi(x)$ in the form of the row (10).

The value of σ_i of each parameter of the each block depends on the physical processes in that block, which are only slightly altered, which the block is running correctly. Then let us assume that the root mean square σ_i does not depend on the block's running time, although it slightly decreases the modelling precision. Furthermore, for each *i*th block we can calculate the initial values of their decrease coefficients α_{i0} from the expression of the type (8).

Due to this, before me start modelling the complex system's of *n* interconnected blocks failure probability changing with the passing of time, we need to create the table of system blocks interconnections, based on the system's topology, and set the dependency of each block's running time t_{ik} from the system's working time. After, for each block, based on its known values of the medium time before failure and its correct working probability from the equation of type (8), we must calculate the initial values of their decrease coefficients α_i, and, by setting their maximal allowed values b_i and the initial expected values m_i, we must calculate their root mean square σ_i from the equation (9) with the usage of the table of values $\Phi(x)$ or the row (10).

In the system, there may be blocks that:

During the system's working time *T* work all the time t_{ik} (table 1).

During the system's working time *T* work episodically (table 2).

Have reserve blocks. In that case the main block during the system's work time *T* works while its failure probability $P_{fi}(t_{ik})$ is lower than the allowed probability P_{afi}, after which we plug in the reserve block and its failure probability $P_{rfi}(t_{rik})$ increases by the exponential law (table 3), where t_{rik} is the reserve block running time.

During the system's working time T are switched on and off depending on the external signal θ_i (table 4).

Table 1.

k	1	2	3	4	5	6	…	K
t_{ik}, h	10^4	$2*10^4$	$3*10^4$	$4*10^4$	$5*10^4$	$6*10^4$	…	T

Table 2.

k	1	2	3	4	5	6	…	K
t_{2k}, h	10^4	10^4	$2*10^4$	$2*10^4$	$3*10^4$	$3*10^4$	…	T
t_{3k}, h	0	10^4	10^4	$2*10^4$	$2*10^4$	$3*10^4$	…	$T-10^4$

Table 3.

k	1	2	3	4	5	6	..	K
P_{f4}	$<P_{mf4}$	$<P_{mf4}$	$<P_{mf4}$	$\geq P_{mf4}$	$\geq P_{mf4}$	$\geq P_{mf4}$	..	$\geq P_{mf4}$
t_{i4}, h	10^4	$2*10^4$	$3*10^4$	$4*10^4$	$5*10^4$	0	..	0
P_{si4}	$<P_{mf4}$	$<P_{mf4}$	$<P_{mf4}$	$<P_{mf4}$	$<P_{mf4}$	$<P_{mf4}$	..	$<P_{mf4}$
t_{si4}, h	0	0	0	0	0	10^4	..	$(K–5)\Delta T$

Table 4.

k	1	2	3	4	5	6	…	K
θ_i	0	0	1	1	0	0	…	1
t_{ik}, h	0	0	10^4	$2*10^4$	$2*10^4$	$2*10^4$	…	$\Delta T(K–1)/2$

Also, during the modelling process the expected values of the blocks' parameters $m_i(t_{ik})$, while changing, will be approaching the dangerous (critical) d_i, that has to be set before the modelling process, and to the maximal allowed value b_i. It is obvious that such situation influences the failure probabilities of those and connected blocks. For instance, the change in the output voltage of the power block leads to the change of the enhancing coefficient of the connected enhancement block. However, the problem of accounting of the blocks' connections influences while computing the failure probabilities still does not have a practically suitable solution [2]. Thus, there are no simple solution to accounting of the connected blocks' failure probabilities, which leads to major errors while computing the complex system's failure probability changing with the passing of time. An analytical accounting of that fact in a complex system, even if we know the required (usually stochastic) dependencies, inevitably leads to complex computations. Thus, we propose a simplified approach to that problem.

We can perform the modelling discretely with some time step ΔT, which is set before the modelling. Thus the system's working time $T = k\Delta T$, where k = 0, 1, …, K, and K is the number of modelling steps, that we set a priori.

During the first modelling step (k = 0) the expected values of the blocks' parameters are known, and during the subsequent step in the time moments $k\Delta T$ we define the working time t_{ik} for each block, and by the equation (8) we compute the probabilities of correct work $P_{ci}(t_{ik})$ and their corresponding failure probabilities $P_{fi}(t_{ik}) = 1 - P_{ci}(t_{ik})$. Then, by the equation (9) with the usage of the table [5] or the approximate value of $\Phi(x)$ we calculate the expected values of the blocks' parameters $m_i(t_{ik})$.

During the modelling, on each step k for each ith block we calculate H values of random parameters ξ_{nih} with the normal distribution and the known excepted value $m_i(t_{ik})$ and the root mean square σ_i. For this, we may calculate each parameter ξ_{nih} by the formula (11):

$$\xi_{ih}^{n} = m_i(t_{ik}) + \sigma_i\left(\sum_{j=1}^{12}\xi_j - 6\right) \quad (11)$$

Where ξ_j is a random number, equally distributed in the interval [0; 1], which we may get using a standard random number generator.

Then we calculate the medium values of those parameters by the formula (12)

$$M_i(t_{ik}) = \left(\sum_{n=1}^{H}(\xi_{ih}^{n})\right)/H \quad (12)$$

If the medium value $M_i(t_{ik})$ of some block's parameters in some time moment t_{ik} falls into a dangerous zone $d_i < |M_i(t_{ik})| < b_i$, we set the connections' coefficients $w(i)$ for this block, that signify the condition of the ith block, and $u(i)$, which signify the ith blocks' proximity to the failed and dangerous block. These parameters may be set as follows:

$w(i)$ = 0 – failed, $w(i)$ = 2 – dangerous, $w(i)$ = 1 – normal

$u(i)$ = 0 – dangerous is farther than over one, $u(i)$ = 1 – dangerous is over one block, $u(i)$ = 2 – dangerous is connected, $u(i)$ = 3 – dangerous is self.

Then we perform the expected value shift according to formula 13:

$$m_i^{*}(t_{ik}) = m_i(t_{ik}) + \sigma_i w(i)u(i)M_i(t_{ik})\mu(M_i(t_{ik})) \quad (13)$$

Where $m^*_i(t_{ik})$ is the shifted expected value of the ith block, $\mu(M_i(t_{ik}))$ is the membership function of the computed expected value to a certain interval, that is defined via the following rules:

if $-\infty < M_i(t_{ik}) < -b_i + m_i(t_{i0})$, then $\mu(M_i(t_{ik})) = 1$

if $-b_i + m_i(t_{i0}) \le M_i(t_{ik}) < -d_i + m_i(t_{i0})$,

then $\mu(M_i(t_{ik})) = \max\begin{Bmatrix}(M_i(t_{ik}) - m_i(t_{i0}) + d_i)/(d_i - b_i); \\ (M_i(t_{ik}) - m_i(t_{i0}) + b_i)/(b_i - d_i)\end{Bmatrix}$

if $-d_i + m_i(t_{i0}) \le M_i(t_{ik}) < m_i(t_{i0})$,

then $\mu(M_i(t_{ik})) = \max\begin{Bmatrix}(M_i(t_{ik}) + m_i(t_{i0}))/d_i; \\ (M_i(t_{ik}) - m_i(t_{i0}) + d_i)/d_i\end{Bmatrix}$

if $m_i(t_{i0}) \le M_i(t_{ik}) < d_i + m_i(t_{i0})$,

then $\mu(M_i(t_{ik})) = \max\begin{Bmatrix}(-M_i(t_{ik}) + m_i(t_{i0}) + d_i)/d_i; \\ (M_i(t_{ik}) - m_i(t_{i0}))/d_i\end{Bmatrix}$

if $d_i + m_i(t_{i0}) \le M_i(t_{ik}) < b_i + m_i(t_{i0})$,

then $\mu(M_i(t_{ik})) = \max\begin{Bmatrix}(M_i(t_{ik}) - m_i(t_{i0}) - b_i)/(d_i - b_i); \\ (M_i(t_{ik}) - m_i(t_{i0}) - d_i)/(b_i - d_i)\end{Bmatrix}$

if $b_i + m_i(t_{i0}) \le M_i(t_{ik}) < \infty$, then $\mu(M_i(t_{ik})) = 1$

Then we set the jth block numbers, that are directly connected to the dangerous block, and for them set the coefficient values $w(j)$ = 1, $u(j)$ = 2, and also make the expected value shift, as in the formula (13), and calculate $\mu(M_j(t_{jk}))$ via the rules 1)-6).

Then we define the numbers of qth blocks, that are connected to the found ith block over a single block, and for them we set $w(q)$ = 1, $u(q)$ = 1, and also make the expected value shift, as if in the formula (13), and calculate $\mu(M_q(t_{qk}))$ via the rules 1)-6)

Now, if on the first modelling step (k = 0) after the excepted value shift we'll have a block, which has its parameter absolute value more than the maximal allowed ($|m_i(t_{ik})| > b_i$), we consider such block failed, its failure probability is set to 1 (P_{fi} = 1), and the whole system's failure probability equals 1 (P_f = 1). Otherwise, we must calculate, based on the shifted expected values, the failure values on all the blocks by the formula (9), which is a simplified equivalent of the conditional probabilities computation. Afterwards we may calculate the failure probability of the whole system, using, for instance, the polynomial formula (6). We may increase the computation precision for each system by tuning the connections' coefficients w and u according to the experimental results.

During the next modelling steps (k > 0) by the computed failure probabilities values and their working time moments from the equations of type (8) we compute the new values of the decrease coefficients α_{ik}. Then, during the next time moment from the equation (8) we compute their failure probabilities, from the equation (9) we compute their expected values $m_i(t_{ik})$, compute new random values, etc.

Thus, in the proposed modelling method we reach the accounting of the connections with the dangerous situation by the discrete changing of the expected value of the current and the connected blocks, which allows accounting the blocks' interconnections impact to the change in their failure probability. We may simplify the procedure even more, if we discretely alter the values of the decrease coefficients α_i.

IV. Example of Modelling the System's Failure Probability Modelling

Let us examine a system of four blocks, which scheme is depicted in the Fig. 1.

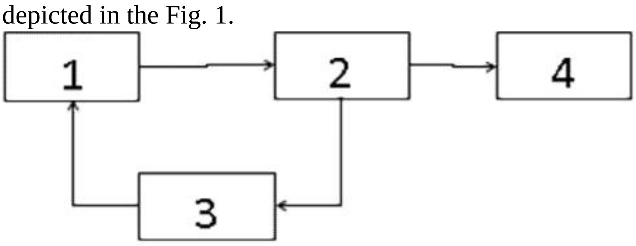

Figure 1. Example system scheme

Let us assume, that all blocks work continuously, and that each block's medium time before failure is 27 000 hours, and the initial failure probability is 0.004. The time step value is 10 000 hours.

After making five test modellings, we depicted the resulting data in the graph seen in the Fig. 2.

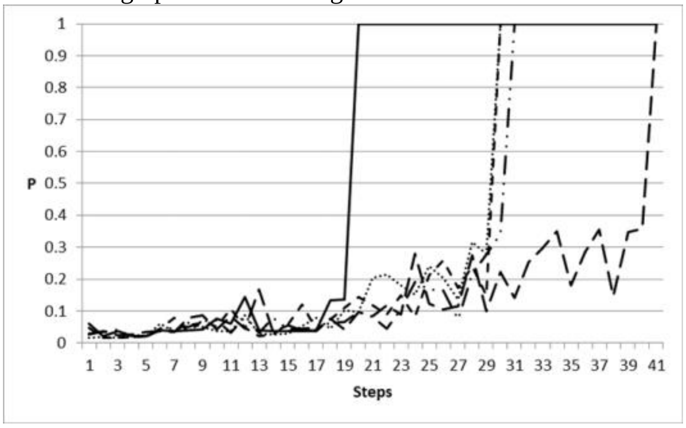

Figure 2. System failure probability modelling

As you can see in the graph, the failure probability prediction in the simplest case is very stochastic, since a single random peak may cause the whole system's instability. In the following chapters, we will show how to control the system's vitality by applying various methods.

V. Summary

The proposed approach to the connections accounting method between the blocks of a complex system while modelling its failure probability with the passing of time allows accounting the impact of the connected blocks, thus increasing the failure probability prognosis.

We implemented the described algorithm as a C# computer program. We may increase the reliability and precision of the modelling by tuning the connection coefficients and the sampling intervals according to the running results and the prediction of the failure of existing systems.

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